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Phase and Group Velocity of Matter Waves

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Abstract:-

This article is about the phase velocity and group velocity of a matter wave and why particle was started to getting associated with wave packet.

Matter wave:-

The dual nature of light was getting accepted because there enough evidence on both the side. In 1924, Louis de Broglie proposed that even a particle has a wave property, guided by the symmetry of nature. Those waves are called matter waves or de Brogile waves. The wavelength of a photon is given by,

A particle will have wave associated with it and wavelength of

$$\lambda = \frac{h}{m\mathbf{v}}$$

Where
λ = wavelength in meters
v = the velocity in meters/sec
m = the mass in kilograms
h = Plancks's constant in J/Hz

Phase velocity :-

It is the wave velocity of a monochromatic wave, velocity with which a definite phase or a particular point in the wave travels. The phase velocity of a wave is given by **u=w/k**, where w= angular velocity and k= wave number of the wave. And phase velocity is also the wave velocity, so it is also given by, **u = frequency × wavelength**.....(1)

Using relativity $E^2= (pc)^2+(m_0c^2)^2$ and equation 1, we get

$$u = c\sqrt{1 + \frac{m_0^2 c^2 \lambda^2}{h^2}}$$

We see that the phase velocity of wave is always greater than. But this doesn't contradict special theory of relativity as no information or signal can be carried by a monochromatic wave. And also, it was clear that velocity of the wave associated with the particle is no way close to the particle velocity. So the phase velocity of the wave can't be the particle velocity.

Group velocity:-

When the different waves of slightly different wavelengths are superposed, it results in a hump at a definite place on the smooth wave by mutual interference of the waves. This hump or the envelope of waves is called a wave packet. The velocity with which the hump or the wave packet travels is called group velocity, v_g.

It is given by v_g= dw/dk. Now, group velocity of a wave associated with a particle is given by

$$\mathbf{v_g = dw/dk = (dw/dE)*(dE/dp)*(dp/dk)}$$
$$\mathbf{= dE/dp,}$$

where E=ℏw and p=ℏk.

Now the value of dE/dp for both relativistic particle and non relativistic particle is v. So, the group velocity of wave associated with the particle is equal to the velocity of the particle. This was a very satisfying result and also it was proved at it is possible to have a wave motion having a particle characteristics at being in a small region of space.

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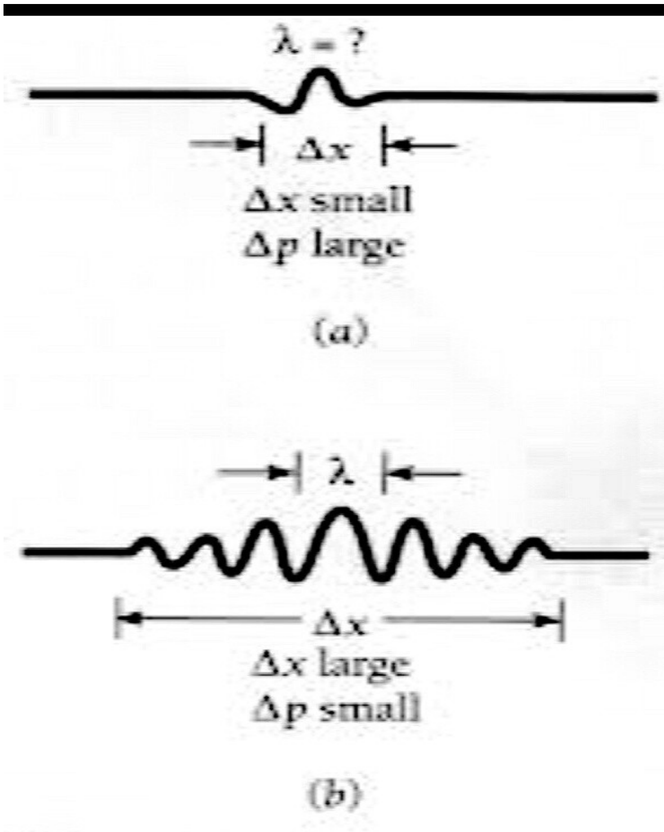
his is of huge significance because it imposed limitations on the precision with which we can calculate position and momentum of a particle simultaneously. As you see, particles can be treated as localized wave formed by superposition of different wave functions. But for a localized wave, the wavelength isn't well defined, but the position is. For example, take a rope and give it sudden jerk. You can see the hump moves and can spot the position but you can't determine the wavelength. But for a normal wave, wavelength is well defined but position isn't as it is spread throughout.



This leads directly to the **uncertainty principle** and to the basics of quantum mechanics.

Reference:-
Modern physics:- AB Gupta.
Quantum mechanics: Griffith
Image1:toppr
Image2:-ocw.nthu.edu.tw