



# Lorentz Transformation

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Einstein in 1905, after a thorough analysis of facts and after the failure of aether theory, proposed the special theory of relativity. It has two postulates.

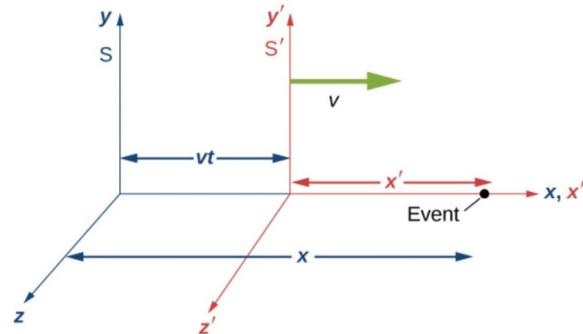
- All the laws of physics, mechanics or electromagnetic, are the same in all inertial frames in relative uniform translation.
- The speed of light in free space is the same in all inertial frames and is independent of the motion of the observer or its source.

Now the first postulate is kind of similar to the Newtonian relativity, but one major problem that arised there was that, it failed to prove the invariance of electromagnetism in Galilean transformation. In an attempt to prove the invariance, aether was introduced. But that didn't last long as Michaelson Morley experiment failed. Einstein included the invariance of electromagnetism in the postulate, which meant, one of the Galilean transformation or the Maxwell's electromagnetic equations need to be modified. Einstein saw no problem in the Maxwell's relations.

The second postulate is the most astonishing and unintuitive about the special theory of relativity. By this postulate Einstein was forced to modify Galilean transformation. The transformation equation that satisfy the relativistic postulate are the **Lorentz transformation**.

**Derivation in short:-** Suppose, S and S' are two reference frames, parallel each other and S' frame is traveling with velocity v w.r.t frame S. At time  $t=t'=0$ , their origin coincide at O.

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A light signal is originated at  $t=0$ . By the postulate of special theory of relativity, the wavefront of the system at time t is given by,

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

c being the velocity of light in free space. In the S' system, the wavefront will be given by,

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0.$$

As the relative motion doesn't change the transverse motion, we can write.

$$x^2 - c^2t^2 = x'^2 - c^2t'^2.$$

We need the mutual transformation to be symmetrical and finite. These requirements can only be met by linear transformation.

$$\begin{aligned} x' &= \gamma(x - vt) \\ x &= \gamma'(x' + vt') \end{aligned}$$



By some substitution and rearrangement we get the required Lorentz transformation.

$$x' = \gamma(x - vt),$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - vx/c^2),$$

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**Discussion:-** Lorentz transformation gets reduced to Galilean transformation when the relative speed is far too less than the speed of light. The beauty of this Lorentz transformation or the special theory of relativity is that, position and time depends on the observer's frame of reference and not the absolute frame of reference as said by the Newtonian mechanics. The Lorentz transformation implies that the events that occur at same position at different times in one frame, can be seen to occur at different places from other frame. So, by this way, a time difference can be completely or partially be transferred into space difference and vice versa, and this revolutionary concept unified space and time into a four dimensional continuum called space time continuum.

**Reference:-**

Modern physics:- AB Gupta.

Image:- Opentextbc