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Complex Numbers

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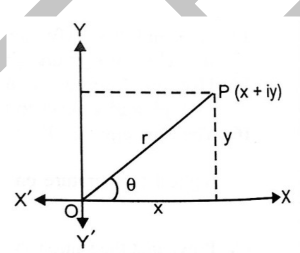
Abstract:

This article is about complex numbers.

A number(z) is called complex number if it is in the form z=p+iq, where p is the real part and q is the imaginary part and $i=\sqrt{-1}$. If p=0 then z becomes purely imaginary and if q=0 then z becomes purely real.

Argand Diagram:

If a complex number is plotted in a xy plane the x axis is called the real axis and the y axis is called the imaginary axis. A point P whose coordinates are (x,y) represents the complex number x+iy. The distance OP is the modulus of (x+iy) and the angle, OP makes with the x axis is the argument of (x+iy).



Addition of Complex Number:

If p+iq and r+is are two complex numbers then (p+iq) + (r+is) = (p+r) + i (q+s)

Subtraction of Complex Number:

If p+iq and r+is are two complex numbers then (p+iq) - (r+is) = (p-r) + i (q-s)

Multiplication:

$(p+iq) \times (r+is) = pr - qs + i(ps+qr)$

Division:

$(p+iq) \div (r+is) = [(pr+qs)/(r^2+s^2)] + i[qr-ps/(r^2+s^2)]$

Conjugate of a Complex Number:

Two complex numbers are called conjugate of each other if they only differ in the sign of the imaginary part. For example if z = p+iq then complex conjugate of $z = z^* = p-iq$.

Types of Complex Number:

1. **Cartesian form:** x+iy
2. **Polar form:** r(cosθ+ i sinθ)
3. **Exponential form:** $re^{i\theta}$

Polar Form(r,θ) of Complex Number:

Let x+iy be a complex number. Putting x= r cosθ and y= r sinθ, we get r= $\sqrt{(x^2+y^2)}$. Therefore, z=r(cosθ+ isinθ).

Then r is called the modulus or the absolute value of the complex number and is denoted by | x+iy |.
The angle θ is called the argument or amplitude of the complex number and is denoted by arg.(x+iy).

De Moivre's Theorem :

$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$

Relation between Circular and Hyperbolic Functions:

A. (1) $\cosh^2 x - \sinh^2 x = 1$, (2) $\text{sech}^2 x = 1 - \tanh^2 x$,

(3) $\text{cosech}^2 x = \coth^2 x - 1$

B. (1) $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

(2) $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

(3) $\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

C. (1) $\sinh 2x = 2 \sinh x \cosh x$ (2) $\cosh 2x = \cosh^2 x + \sinh^2 x$

(3) $\cosh 2x = 2\cosh^2 x - 1$ (4) $\cosh 2x = 1 + 2\sinh^2 x$

(5) $\tanh 2x = \dfrac{2 \tanh x}{1 + \tanh^2 x}$

D. (1) $\sinh x + \sinh y = 2 \sinh \dfrac{x+y}{2} \cosh \dfrac{x-y}{2}$

(2) $\sinh x - \sinh y = 2 \cosh \dfrac{x+y}{2} \sinh \dfrac{x-y}{2}$

(3) $\cosh x + \cosh y = 2 \cosh \dfrac{x+y}{2} \cosh \dfrac{x-y}{2}$

(4) $\cosh x - \cosh y = 2 \sinh \dfrac{x+y}{2} \sinh \dfrac{x-y}{2}$

Note: For proof, put $\sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

Formula for Hyperbolic Functions:

$\sin ix = i \sinh x$
$\cos ix = \cosh x$
$\tan ix = i \tanh x$

$\sinh ix = i \sin x$
$\cosh ix = \cos x$
$\tanh ix = i \tan x$

Circular Functions of Complex Number:

$\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

If θ = z, then $\cos z = \dfrac{e^{iz} - e^{-iz}}{2}$ and $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}$

Hyperbolic Functions:

(i) $\sinh x = \dfrac{e^x - e^{-x}}{2}$ (ii) $\cosh x = \dfrac{e^x + e^{-x}}{2}$ (iii) $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(iv) $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ (v) $\text{sech}\, x = \dfrac{2}{e^x + e^{-x}}$ (vi) $\text{cosech}\, x = \dfrac{2}{e^x - e^{-x}}$

(vii) $\cosh x + \sinh x = \dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = e^x$

(viii) $\cosh x - \sinh x = \dfrac{e^x + e^{-x}}{2} - \dfrac{e^x - e^{-x}}{2} = e^{-x}$

(ix) $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$.

Reference:

Mathematical physics: HK Dass and Dr. Rama Verma.